Effects of Electrode Inertia on Vibration of Piezoelectric Plate with Dissipation

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***Abstract*—In this study, we analyze the thickness-shear vibration of a simple resonator model by means of complex material constants with consideration of the electrode inertia. The effects of viscosity on frequency behavior of piezoelectric plate are presented and discussed. The results can be useful for design of the relevant resonators.**

I. INTRODUCTION

It was well known that piezoelectric materials are widely used in resonators, sensors and transducers. Almost all the modeling and analysis of wave propagation are based on the linear theory of piezoelectricity with quasi-electrostatic approximation, where the medium is assumed to be perfectly elastic and perfectly insulating to electric current. As operating frequencies get higher and sizes of acoustic wave devices become smaller, the dissipation of energies becomes more significant and should be considered in more realistic theories or modeling.

As presented in [1-2], an approach to treat with viscous dissipation of piezoelectric material is to assume that elastic modulus, piezoelectric coefficient, and dielectric permittivity coefficient are all complex constants, so the attenuation and other effects of the viscosity on the structural vibrations or the propagation of waves can be considered and analyzed. Numerous investigations of damping in solid represented by viscoelastic model have been undertaken for the isotropic materials by researchers in various disciplines because of its important applications [3-6]. However, there are few studies using viscoelastic models for vibrations of piezoelectric solids with consideration of electrode inertia.

In resonator manufacturing, one electrode is deposited first with a pre-determined thickness. Then the electrode on the other side of the piezoelectric plate has a thickness that is determined by the desired frequency of the electroded plate. If the electrodes on the crystal plate are very thin, their mechanical effects can be neglected. When the electrodes are not very thin, their mechanical effects, e.g. inertia and stiffness, need to be considered.

In this study, we investigate the inertial effect of the electrode mass with consideration of dissipation of the piezoelectric plate.

II. PROBLEM FORMULATION

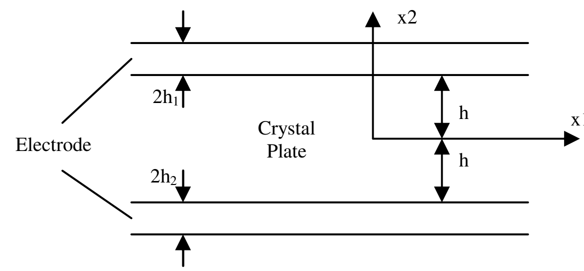

Figure 1. A crystal plate with electrodes

Consider a plate of piezoelectric materials with electrodes of unequal thickness as shown in Fig.1. The electrodes are shorted. The moving and the charge equations are given as follows:

$$T_{ji,j} = \rho \ddot{u}_i, \dot{D}_{i,i} + J_{i,i} = 0, i, j = 1,2,3, \ |x_2| < h, \tag{1}$$

and the constitutive equations with consideration of dissipation are given as[1-2]

$$\begin{aligned} &T_{ij} = c_{ijkl}S_{kl} - e_{ijk}E_k + \eta_{ijkl}\dot{S}_{kl}, \\ &D_i = e_{ijk}S_{jk} + \varepsilon_{ik}E_k, \qquad |h_2| < h, \\ &J_i = \sigma_{ik}E_k, \end{aligned} \tag{2}$$

where S_{ij} and E_i are the strains, and electric fields, respectively; c_{ijkl}, η_{ijkl} and ε_{ij} are the elastic, viscosity, and dielectric permittivity coefficients, respectively; e_{ij} is the piezoelectric coefficients; σ_{ij} is the electric conductivities. The subscript comma denotes a partial derivative with respect to the coordinates and a superimposed dot represents the derivative with respect to time. The extended geometric relations are

$$S_{ij} = \left(u_{i,j} + u_{j,i}\right)/2, \quad E_i = -\phi_{,i}, \ |x_2| < h, \tag{3}$$

where ϕ is the electric potential. The subscript comma indicates the differential with respect to the coordinates. We here study the special case of thickness-shear vibration of a piezoelectric plate. Consider an unbounded, rotated Y-cut

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crystal plate with two electrodes. For monoclinic crystals, thickness-shear modes described by the following displacement and potential fields are allowed:

$$u_1 = u_1(x_2, t),\ u_2 = u_3 = 0,\ \phi = \phi(x_2, t), \tag{4}$$

The moving equations and charge equation of quartz plate are reduced as:

$$T_{2i,2} = \rho \ddot{u}_i,\ \dot{D}_{2,2} + J_{2,2} = 0, \tag{5}$$

The non-trivial components of strain, electric field, stress, and electric displacement are

$$2S_{12} = u_{1,2},\ E_2 = -\phi_{,2}, \tag{6}$$

and

$$\begin{aligned} &T_{31} = c_{56}u_{1,2} + e_{25}\phi_{,2} + \eta_{56}\dot{u}_{1,2},\quad T_{12} = c_{66}u_{1,2} + e_{26}\phi_{,2} + \eta_{66}\dot{u}_{1,2},\\ &D_2 = e_{26}u_{1,2} - \varepsilon_{22}\phi_{,2},\ D_3 = e_{36}u_{1,2} - \varepsilon_{23}\phi_{,2}, \end{aligned} \tag{7}$$

Substituting (7) into (5), we can obtain the equation of motion and the charge equation

$$\begin{aligned} &T_{21,2} = c_{66}u_{1,22} + e_{26}\phi_{,22} + \eta_{66}\dot{u}_{1,22} = \rho \frac{\partial^2 u_1}{\partial t^2},\\ &D_{2,2} = e_{26}\dot{u}_{1,22} - \sigma_{22}\phi_{,22} - \varepsilon_{22}\dot{\phi}_{,22} = 0, \end{aligned} \tag{8}$$

Boundary conditions are written as:

$$\begin{aligned} &-T_{2j} = 2\rho_1 h_1 \ddot{u}_j,\ x_2 = h,\\ &T_{2j} = 2\rho_1 h_2 \ddot{u}_j,\ x_2 = -h,\\ &\phi(x_2 = h) = \phi(x_2 = -h), \end{aligned} \tag{9}$$

III. SOLUTIONS OF THE PROBLEM

We assume the following displacement and potential fields:

$$\begin{aligned} &u_1 = u_1(x_2)\exp(i\omega t),\ u_2 = u_3 = 0,\\ &\phi = \phi(x_2)\exp(i\omega t), \end{aligned} \tag{10}$$

Substituting (10) into (8) we can obtain

$$\begin{aligned} &c_{66}u_1'' + e_{26}\phi'' + i\omega\eta_{66}u_1'' = -\rho\omega^2 u_1,\\ &i\omega e_{26}u_1'' - \sigma_{22}\phi'' - i\omega\varepsilon_{22}\phi'' = 0. \end{aligned} \tag{11}$$

Assuming

$$b = \frac{i\omega e_{26}}{\sigma_{22} + i\omega\varepsilon_{22}},$$

we can obtain the follows

$$\begin{aligned} &u_1'' + b_1^2\omega^2 u_1 = 0,\\ &\phi'' = bu'', \end{aligned} \tag{12}$$

where $b_1^2 = \rho / (c_{66} + i\omega\eta_{66} + be_{26})$.

From (12), we can obtain displacement and potential

$$\begin{aligned} &u_1 = A_1\sin(b_1\omega x_2) + A_2\cos(b_1\omega x_2),\\ &\phi = bu_1 + B_1x_2 + B_2. \end{aligned} \tag{13}$$

and stress (harmonic factor is omitted)

$$\begin{aligned} T_{21} = &A_1\left[(c_{66} + i\omega\eta_{66})b_1\omega + e_{26}bb_1\omega\right]\cos(b_1\omega x_2)\\ &-A_2\left[(c_{66} + i\omega\eta_{66})b_1\omega + e_{26}bb_1\omega\right]\sin(b_1\omega x_2) + B_1e_{26}, \end{aligned} \tag{14}$$

Substituting (13)-(14) in to boundary condition (9), we can arrive at

$$\begin{aligned} &-A_1\left[(c_{66} + i\omega\eta_{66})b_1\omega + e_{26}bb_1\omega\right]\cos(b_1\omega h)\\ &+A_2\left[(c_{66} + i\omega\eta_{66})b_1\omega + e_{26}bb_1\omega\right]\sin(b_1\omega h) - B_1e_{26}\\ &= -2\rho_1 h_1\omega^2\left[A_1\sin(b_1\omega h) + A_2\cos(b_1\omega h)\right], \end{aligned} \tag{15}$$

$$\begin{aligned} &A_1\left[(c_{66} + i\omega\eta_{66})b_1\omega + e_{26}bb_1\omega\right]\cos(b_1\omega h)\\ &+A_2\left[(c_{66} + i\omega\eta_{66})b_1\omega + e_{26}bb_1\omega\right]\sin(b_1\omega h) + B_1e_{26}\\ &= -2\rho_1 h_1\omega^2\left[-A_1\sin(b_1\omega h) + A_2\cos(b_1\omega h)\right], \end{aligned} \tag{16}$$

$$bA_1\sin(b_1\omega h) + B_1h = 0, \tag{17}$$

In order to obtain the nontrivial solutions of the above-mentioned unknown constants A_1, A_2, B_1, the determinant of the coefficient matrix of these linearly algebraic equations must equal zero. So the frequency equation can be obtained.

IV. RESULTS AND DISCUSSION

We calculated the frequency for different piezoelectric materials and gold or silver electrodes with consideration of dissipation of piezoelectric plate. The necessary constants of materials in calculation can be found in [7].

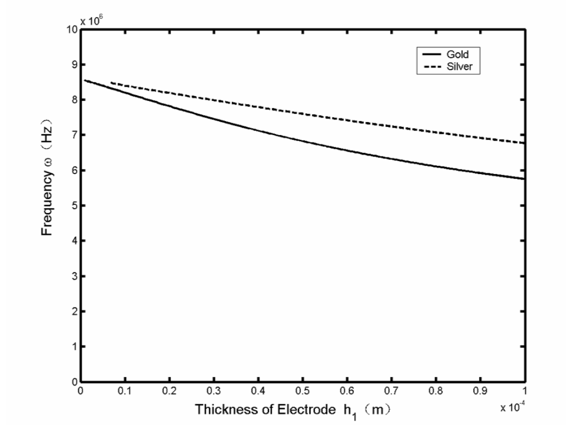

Figure 2. Frequency versus thickness of electrodes for $BaTiO_3$

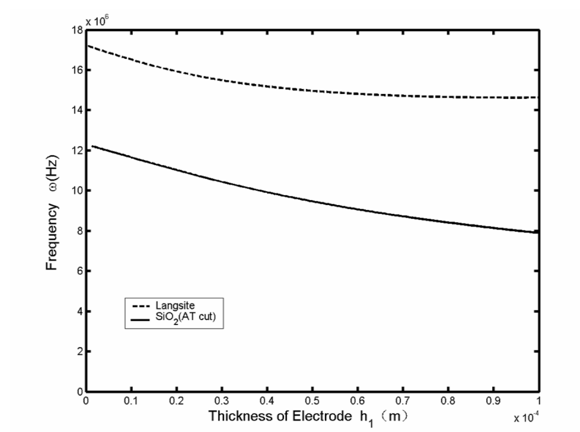

Figure 3. Frequency versus thickness of electrodes for Langasite or SO_2

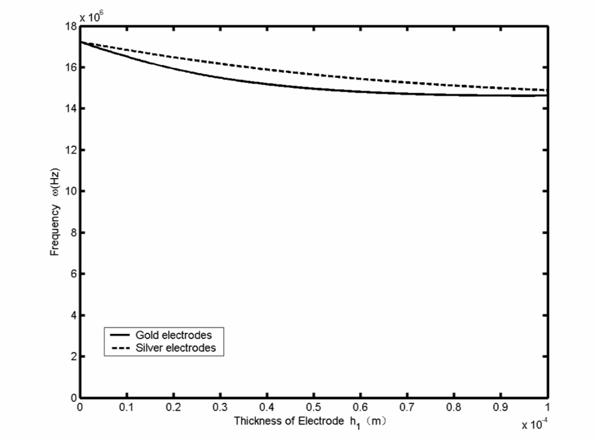

Figure 4. Frequency versus thickness of electrodes for langasite

Fig.2 shows the effect of thickness of electrode on the frequency with viscosity $\eta_{66} = 43\mathrm{N \cdot s/m^2}$ and electric conductivity $\sigma_{22} = 9.872\times10^{-9}(Ohm\cdot m)^{-1}$ for gold and silver electrodes. We can find that the gold electrode has more remarkable effect than the silver electrode on the frequency for same viscous dissipation because gold is heavier than silver.

Fig.3 presents the frequency versus thickness of electrodes for langasite and quartz. The viscous coefficient of quartz is $\eta_{66} = 0.32\times10^{-3} N\cdot s/m^2$, the electric conductivity is $\sigma_{22} = 0.005\times10^{-12}(Ohm\cdot m)^{-1}$. The viscous coefficient of langsite is assumed as $\eta_{66} = 0.64\times10^{-3} N\cdot s/m^2$, the electric conductivity is $\sigma_{22} = 0.01\times10^{-12}(Ohm\cdot m)^{-1}$. It can be seen that the frequency of langasite plate is higher than that of the quartz plate. Fig.4 indicates the effect of gold or silver electrodes on the frequency for langasite.

From the results we can find, for piezoelectric crystal, such as quartz or langasite, the viscosity is always so small that the effect on the frequency can be neglected, but the frequency with consideration of dissipation provides a method to calculate the electric parameters. For piezoelectric ceramics, such as $BaTiO_3$, the viscosity has a remarkable on the frequency.

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